Teaching Reform Practice of Security Investment Course for Financial Mathematics Undergraduate

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Keywords: Security Investment, Practice Teaching Method

Abstract: The security investment is a comprehensive course with a combination of theories and practices. This paper tries to give some thoughts of reforming the teaching of security investment for the undergraduates majored in financial mathematics. Based on the experiences of teaching this course in Sustech, we emphasize both the mathematical and technical details and practical teaching. We give an illustration of the mathematical derivation for Markowitz minimum variance set. The experience of practical teaching in Sustech are summarized. Through trading competition and course projects, the students could consolidate the investment theories and improve their world problems solving abilities. Finally, future improvements in teaching this course are discussed.

1. Introduction

Security investment is a core course for undergraduate students majored in financial mathematics or other disciplines related to finance. After taking this course, the students should have a basic understanding of the modern portfolio theories, including Markowitz's portfolio selection model, the capital asset pricing model and the arbitrage pricing theory. They should also understand the idea of risk-return trade-off and the theory of efficient market hypothesis [1]. In the security investment course, the introduction of the financial markets will be covered and the common financial instruments such as stocks, bonds and derivatives will be studied. Besides the theoretical parts, the applications are also should be emphasized. Students would learn how to apply the investment theories to analyse the investment opportunities, manage portfolios, reduce the risk and reach their financial goals.

Due to the characteristic of the security investment course and the requirements of financial mathematics major, this paper will discuss the teaching reform of the security investment course, so that the financial mathematics student could be more competitive in the fast-changing world.

2. Strengthen the Teaching of Mathematical Derivation

In nowadays, especially in China, more and more financial jobs require candidates with advanced degrees, therefore, most of the financial mathematics undergraduates intend to pursue graduate level studies either in home country or overseas. To success in applying graduate programs and their future graduate studies and research, the students must have a very solid background in mathematics and equipped with strong quantitative skills.

The current popular textbooks of security investments for undergraduates have concentrated on providing intuition of financial markets and skip most of the mathematical and technical detail [1], which are essentially important for math major students. Therefore, when teaching this course for the financial mathematics students, we supplement many mathematical derivations for some important parts of investment theories.

Here we'll give an example to show the process in detail. When introduce the Markowitz's portfolio selection model, the students are told the minimum-variance set of risky assets when short selling allowed is a hyperbola in the $E(r) - \sigma$ plane. But in most textbooks, the mathematical

  DOI: 10.25236/iche.2022.006

details are not given. During our lectures, we first set up the mathematical framework for this the minimum-variance set problem as the following:

Consider n risky assets, the mean (or expected) rates of return are $E(r_1), E(r_2), \cdots, E(r_n)$ and the covariance matrix is Σ. A portfolio is defined by a set of n weights w_i, $i = 1,2,\cdots,n$, that sum to 1.

To determine the portfolio on the minimum variance frontier, we solve the following optimal problem [3]:

$$\min \frac{1}{2} W^T \Sigma W \tag{1}$$

subject to

$$W^T \mathbf{r} = r_0,\ W^T \mathbf{1} = 1 \tag{2}$$

where $W = [w_1 \quad w_2 \quad \cdots \quad w_n]'$, $\mathbf{r} = [E(r_1) \quad E(r_2) \quad \cdots \quad E(r_n)]'$, r_0 is the target expected return and $\mathbf{1} = [1 \ 1 \ \cdots \ 1]'$.

Let

$$L(W,\lambda,\mu) = \frac{1}{2} W^T \Sigma W + \lambda(r_0 - W^T \mathbf{r}) + \mu(1 - W^T \mathbf{1}) \tag{3}$$

The Lagrange Multiplier method gives following $n+2$ equations.

$$\begin{cases} \dfrac{\partial L}{\partial w_1} = \displaystyle\sum_{j=1}^{n} w_j \sigma_{1j} - \lambda E(r_1) - \mu = 0 \\ \qquad\qquad \vdots \\ \dfrac{\partial L}{\partial w_n} = \displaystyle\sum_{j=1}^{n} w_j \sigma_{nj} - \lambda E(r_n) - \mu = 0 \end{cases} \tag{4}$$

$$\begin{cases} \displaystyle\sum_{i=1}^{n} w_i E(r_i) = r_0 \\ \displaystyle\sum_{i=1}^{n} w_i = 1 \end{cases} \tag{5}$$

Put them in matrix form, we have

$$\begin{cases} \mathbf{1}^{\mathrm{T}} W = \mu\left(\mathbf{1}^{\mathrm{T}} \Sigma^{-1} \mathbf{1}\right) + \lambda\left(\mathbf{1}^{\mathrm{T}} \Sigma^{-1} \mathbf{r}\right) = 1 \\ \mathbf{r}^{\mathrm{T}} W = \mu\left(\mathbf{r}^{\mathrm{T}} \Sigma^{-1} \mathbf{1}\right) + \lambda\left(\mathbf{r}^{\mathrm{T}} \Sigma^{-1} \mathbf{r}\right) = r_0 \end{cases} \tag{6}$$

Then solve the above linear systems about μ and λ by Cramer's rule, we have

$$\mu = \frac{\begin{vmatrix} 1 & b \\ r_0 & c \end{vmatrix}}{\begin{vmatrix} a & b \\ b & c \end{vmatrix}} = \frac{c - r_0 b}{ac - b^2} = \frac{c - r_0 b}{\Delta}, \lambda = \frac{\begin{vmatrix} a & 1 \\ b & r_0 \end{vmatrix}}{\begin{vmatrix} a & b \\ b & c \end{vmatrix}} = \frac{a r_0 - b}{ac - b^2} = \frac{a r_0 - b}{\Delta} \tag{7}$$

where $a = \mathbf{1}'\Sigma^{-1}\mathbf{1},\ b = \mathbf{1}'\Sigma^{-1}\mathbf{r},\ c = \mathbf{r}'\Sigma^{-1}\mathbf{r},\ \Delta = ac - b^2$.

We could use the Cauchy-schwarz inequality to show that $\Delta > 0$.

Then for the portfolio P on the minimum variance frontier with standard deviation σ_0 and expected return r_0,

$$\sigma_0^2 = W^T \Sigma W = W^T(\lambda \mathbf{r} + \mu \mathbf{1}) = \lambda(W^T \mathbf{r}) + \mu(W^T \mathbf{1})$$

$$\sigma_0^2 = \frac{ar_0 - b}{\Delta} r_0 + \frac{c - r_0 b}{\Delta} = \frac{ar_0^2 - 2br_0 + c}{\Delta} \tag{8}$$

which is exactly a hyperbola in the $E(r) - \sigma$ plane. This example is also a good illustration of integrating the calculus techniques, matrix theory and probability theory into the teaching of security investment course. Besides learning new materials, the students also have a chance to review the math skills.

We also proved the two-fund theorem and one-fund theorem. We adopted the proofs from [2], and use the method of solving linear equations to find optimal risky portfolio instead of just listing the complicate formula to students which is very hard to understand and memorize.

Through these rigorous processes, we not only provide the proof of the theoretical parts, but also provide the algorithms so that later students could apply them to construct the efficient frontier and optimal asset allocation when using real world data.

Besides giving the mathematical details in the classroom teaching, we also provide students a lot of mathematical derivation problems in their homework assignments to deepen their understanding of the course material.

3. Increase the Proportion of the Practice Teaching

The application of investment strategies also plays an important role in the teaching of security investment. The students should improve their practical abilities which are helpful for them to succeed in their future career in the finance industry. Several researchers have studied the practice teaching in investment courses [4,5,6]. This study would give some thoughts of practice teaching based on the experiences from teaching this course in Sustech.

3.1. Stock Trading Competition

Few of the students have investment experiences with securities such as stocks, bonds and derivatives. The best way to help the students to get familiar with the financial markets is to get them immersed in the market. At the beginning of the semester, we start a trading competition among the students who are taking this course. There are a bunch of online stock trading simulators. We used the Investment Training Platform offered by The China Stock Market & Accounting Research (CSMAR) since 2016. The students get the same amount of virtual money to start with. They could trade all the exchange traded products in China financial markets and the products from HKEX through the platform in real time. We evaluate the student's trading performance based on both the profit/loss and risk criterions. At the end of the competition, we will give extra bonus points to the students ranked top 6 or so. In this way, the students are encouraged to actively participate in the competition. After the competition, the students will be asked to write a summary report of their trading process and share their experiences they have learned from this virtual trading.

Besides the trading competition organized in class, the students are encouraged to participate in the competition sponsored by other parties, such as China (Hengqin) International Universities Quantitative Finance Competition.

3.2. Class Projects

Besides the written homework assignments, we also assign at least two class projects to train students linking the theory with real world problems. Usually, the first project is the implementation of Markowitz's Portfolio Optimization Model for stock prices data and a broad market index data of China market. The students first need to collect market data through the professional data providers like Wind Financial Terminal, which is widely used in China financial industry. And then the students compare their results from the Markowitz's framework with the optimal risky portfolio formed from the index model. Through their own practice, the students could understand the procedure of portfolio construction better and can find problems which is not covered in the

textbook and the in-class lectures. For instance, they could not always find the optimal risky portfolio for real data since the minimum variance set is a parabola. If the risk-free rate is even higher than the expected return of the global minimum variance portfolio, then the line through the risk-free asset tangent to the minimum variance set will be under the global minimum variance portfolio.

For the second project, the students are free to choose any topic related to the investment theories and perform empirical studies and write a report. Some students studied the application of the factor models. Some students studied the efficiency of China financial markets. Some of them even choose topics outside of textbook. This could also improve the students' self-learning abilities and research skills. Because of the progress in data science and financial technology (fintech), we also encourage the students to develop their own quantitative investment strategies. The students who succeeded in building their quantitative trading models are awarded with extra bonus points for their final grades as well. These trainings also benefit students with their later thesis writing. For instance, quite a lot of students worked on applications of machine learning models on quantitative trading for their graduation thesis.

3.3. More Examples

Most of the students are not familiar with the investment theories and financial instruments. When explaining those concepts, we could give students more real financial stories relating to the concepts. For instance, when we discuss the risk of buy or sell on margin, BOC crude oil treasure product incident could be a good example. With more lessons from the real world, the students could master the financial concepts better and get more hands-on experiences.

4. Future Improvements

Security investment theories and applications change with the development of the technologies and financial environments. The teaching and study of security investments are full of challenge for both the students and the instructors. For better quality of teaching, we could improve our teaching in the following ways.

First, the instructors could attend more financial industry forums or cooperate with experts from industry to gain more latest practical investment strategies and ideas and convey them to the students.

Secondly, we could invite professionals from financial institutions to give small lectures on the practical investment strategies and find opportunities of real projects from the financial markets for the students to practice.

Thirdly, we could ask feedbacks from the students frequently and adjust our teaching materials and methods accordingly.

Acknowledgements

This research is funded by SUSTech Teaching Reform Project ("The practice of quantitative investment in teaching of security investment course for undergraduate", Grant No. Y01051946).

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